Management and Economic Journal

Dependence and Risk Contagion between MENA Stock Markets and FX Returns: An Application Based On Extreme Value Copula Model

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Abstract *:- This paper proposes a method for modeling and estimating the dependence between MENA stock markets and FX returns based on copula and extreme value theory. Each return is modeled by GARCH models with the joint distribution of innovations modeled by copulas. Generalized Pareto distribution is adopted to model marginal distributions both in the left and in the right tail. Our empirical results suggest that the majority of pairs of daily financial returns (stock market return /FX return) seem better modeled by the bivariate Clayton copula during the two periods of study. In addition, all estimated parameters are positive reflecting a positive dependence between these returns during periods of declining and rising market. However, the intensity of this dependence differs from one country to another and from one period to another and the released results depend on the selected copula. Our findings have important implications for regional investors opting for intra-regional diversification strategy by taking into account joint tail risk and providing recommendations to clarify the influence of risk on the investor and especially to optimize investment choices.*

Keywords: *Subprime financial crisis, stock market return, Foreign exchange return, Extreme value theory, Copulas.*

1. Introduction :

Rare yet extreme events such as the financial crises, Crashes and the oil crises are among the most surprising phenomena of Finance. However, the economists do not highlight these events, whose effects concern as well the individual investors and the institutions as the financial system in general. Therefore, the modeling of these events is now becoming a particularly active framework of research today. In particular, since a few years, we note an interest growing for the application of Extreme value theory (EVT) for the modeling of such events. Fields of application using EVT techniques have grown steadily during these last years, reaching various areas.

In hydrology, domain in which flood forecasting is particularly important [Davison and Smith, 1990; Katz, 2002]. In human and social sciences, the extreme value theory can help in the understanding of numerous social problems. For example, the debate, in the field of the demography, which was introduced by Gumbel [1937], and in whom Fréchet took an active part on the notion of "extreme duration of the human life" and on its measure [Thatcher, 1999]. In insurance, the TVE contributes to the understanding of the big disasters [McNeil and al ., (1997); Rootzen and Tajvidi (1997)].

Finally, in Finance, this theory, introduced by Embrechts and al. (1997), McNeil (1998), Longin (1998, 2000); Embrechts (1999); Fernandez (2000); coles (2001); Reiss and Thomas (2001) as well as Gençay and Selçuk (2004), analyzes extreme risk in the financial markets, due to currency crises, stock market shocks and credit failures. Therefore, it establishes an immediate answer to the questioning of the hypothesis of normality especially with high frequency observations.

Thus, the introduction of the EVT became a requirement in several domains that the traditional probability techniques, based on the hypothesis of normality and developed in a gaussian universe, remain unsuitable for the apprehension of this extreme behavior. Indeed, most of the empirical studies and models concern only the "average" properties of probability distributions. In spite of these events are of a considerable importance of the fact that they are connected with the risks of defects of the investors, the risk of bankruptcy of the financial institutions and with the systematic risk, there is relatively little attention paid to these extreme co-movements.

The study of extreme values refers to the branch of statistics that deals with extreme deviations from the average of a probability distribution. It provides a solid framework to study the behavior of extreme events. Thereby it focuses directly on the tails of function distributions. In this sense, we can consider that the extreme value theory is the counterpart of the classical statistical theory; it focuses primarily on the study of extreme returns rather than the entire distribution. Therefore, it could have better results than other approaches in terms of prediction of unexpected extreme changes.

The purpose of this paper is to model explicitly the dependence of extreme co-movements between MENA stock markets and FX returns during the subprime crisis. We combine two models, which are the extreme value theory and the copula approach to have a joint EVT -Copula model with possibly fat tailed return innovations and non-linear property in order to reach a new copula family, called extreme copula.

The remainder of the paper is structured as follow. Section 2 presents an overview of the theoretical framework of the extreme value theory (EVT) and the copula approach. Section 3 outlines the methodology used. Section 4 presents the data and discusses the empirical results. The final section concludes.

2. THEORETICAL BACKGROUND :

2.1. EXTREME VALUE THEORY:

This theory appeared between 1920 and 1940 thanks to Fréchet, Fisher and Tippet, Gumbel and Gnedenko. EVT has two principal models: the block maxima model (BMM) and peak over threshold model (POT). Through the block maxima method, the asymptotic distribution of a series of maxima (minima) is modeled and the distribution of the standardized maximum belongs to one of these extreme value distributions namely Gumbel, Fréchet or Weibull distributions. The standard form of these three distributions is the generalized extreme value distribution (GEV). The peak over threshold method identifies extreme observations that exceed a given threshold u and the limiting distribution of these "exceedances" is a generalized Pareto distribution (GPD).

2.1.1. THE GENERALIZED EXTREME VALUE DISTRIBUTION (GEV) AND BLOCK MAXIMA METHOD:

Suppose that X_t, t=1,2, …n is a sequence of independently and identically distributed random variables and F(x) = Pr $\{X_t \leq x\}$ is a common distribution function. Denote the sample Maxima of X_t by $M_{1=}X_1$; $M_n = \max(X_1, \dots, X_n)$; n≥ 2.

Let IR denote the real line. Given a sequence of $C_n > 0$, $d_n \epsilon$ IR and come non-degenerate distribution function H such that $C_n^{-1}(M_n$- $d_n) \rightarrow H$, so H is one of the three limit distributions:

$$\Lambda(x) = e^{-e^{-x}}, \quad x \in IR$$

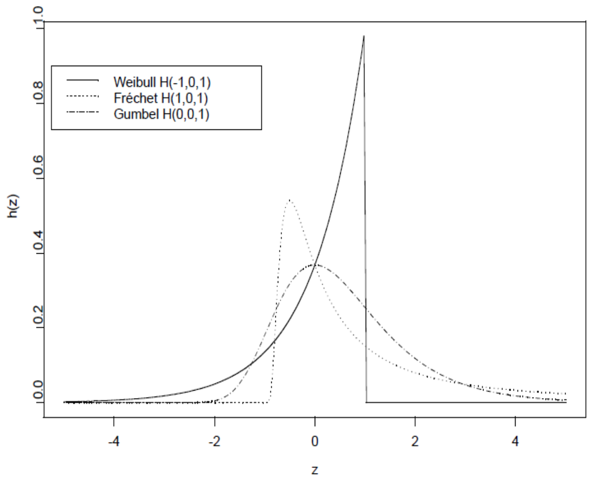
(Gumbel)

$$\Phi_{\alpha}(x) = \begin{cases} e^{-x^{-\alpha}}, & x>0 \\ 0, & x \leq 0 \end{cases}$$

(Fréchet)

$$\psi_{\alpha}(x) = \begin{cases} e^{-(-x)^{\alpha}}, & x \leq 0 \\ 1 \; si & x > 0 \end{cases}$$ (Weibull)

FIG. Density of the extreme values; with $\zeta = 1$ for the Fréchet, $\zeta = 0$ for the Gumbel and $\zeta = -1$ for Weibull distributions



Source : Bechir Raggad : « Fondements de la théorie des valeurs extrêmes. Ses principales applications et son apport à la gestion des risques du marché pétrolier », 2009

2.1.2. Generalized Pareto distribution and peak over threshold (POT):

Let be a series of observationsZ_1, … Z_n following an unknown distribution function F iid. Consider Z_0 the finite or infinite upper limit of the F distribution and N_μ is the number of strictly positive extreme exceeding a high threshold u.

Let us suppose X_i= Z_i -u sample excesses X_1, … , X_n, supposed iid.

The basic idea consists in choosing a sufficiently high level and studying the excess over the threshold. We define then the excess distribution function above the threshold u by:

$$F_u(x) = P\{Z - u \leq x | Z > u\} = P\{X \leq x | Z > u\} = \frac{F(x+u) - F(u)}{1 - F(u)},$$

for x ≥ 0

$F_u(x)$ is the probability that extreme exceeds the threshold u less than or equal to x, knowing that the threshold is exceeded. The essential exponential modeling of excess is the Generalized Pareto (Generalized Pareto Distribution), noted GPD and it is defined by the following distribution function:

$$G_\zeta(x) = \begin{cases} 1 - (1 + \zeta x)^{-1/\zeta} & si\ \zeta \neq 0 \\ 1 - \exp(-x) & si\ \zeta = 0 \end{cases}$$

The GPD brings together three special distributions according to the values of a parameter: When $\zeta > 0$, this is the usual Pareto; when $\zeta < 0$ we are in the presence of Pareto II; and if $\zeta = 0$, we have the exponential law.

2.2. COPULAS MODELS:

Copulas are multivariate distribution functions with standard uniform marginal distributions. Am-dimensional copula is represented as follows:

$$\mathbf{C\ (u) = C\ (}u_1, \ldots, u_m\mathbf{)}$$

Where $u_1, \ldots, u_m$are standard uniform marginal distributions.In such a context, copulas can be used to link margins into a multivariate distribution function. The copula function extends the concept of multivariate distribution for random variables which are defined over [0,1]. This is possible due to the Sklar (1959) theorem that states that copulas may be constructed in conjunction with univariate distribution functions to build multivariate distribution functions.

Sklar's Theorem: Let F_{XY} be a joint distribution function with margins F_X and F_Y. Then there exists a copula C such that for all x, y in R,

$$\mathbf{C}\ (u_x, u_y) = \mathbf{C}\ (F_x(\mathrm{x}), F_y(\mathrm{y}))$$

$$= \mathbf{F}\ (F_x^{-1}(u_x), F_y^{-1}(u_y))$$

$$\mathbf{C}\ (u_x, u_y) = \mathbf{F}\ (\mathrm{x, y})$$

If $F_X$and F_Y are continuous, then C is unique; otherwise, C is uniquely determined on Ran F_X×Ran F_Y and C is invariant under strictly increasing transformations of the random variables.

Here, we study three copulas models with different dependence structure that allow the dependence between extreme events including Student copula, Gumbel and Clayton copula to specify the level and structure of dependence between our extreme financial returns. Indeed, the Student copula presents both right and left tails. Gumbel Copula, meanwhile, apprehends only positive dependences and has the characteristic to represent risks that the dependence structure is more marked on the upper tail. It is for this reason particularly adapted as such in finance to study the impact of the occurrence of high intensity events on the dependence between financial assets. Clayton copula allows modeling only the positive dependences. However, it is intended to reflect a reliance on low-intensity events.

- **The Student- t copula:**

The Student-t copula is defined by:

$$\boldsymbol{C_T(u_1,\dots,u_m) = T_{v,m,\Sigma}\left(T_v^{-1}(u_1),\dots,T_v^{-1}(u_m)\right)}$$

Where $T_{v,m,\Sigma}$ is the multivariate student distribution function with a degree of freedom v and variance-covariance matrix $\sum$.

- **The Clayton Copula:**

$$C(u, v, \theta) = (u^{-\theta} + v^{-\theta} - 1)^{\frac{-1}{\theta}} \quad \text{where } \theta > 0$$

- **The Gumbel Copula:**

$$C(u, v, \theta) = \exp\left[-[(-\text{Ln}(u))^{\theta} + (-\text{Ln}(v))^{\theta}\right]]^{\frac{1}{\theta}} \quad \text{where } \theta \geq 1$$

Where the two variables, u and v, are cumulative distribution functions. The parameter θ measures the degree of dependence between risks.

3. METHODOLOGY:

First, we adopt the Generalized Pareto Distribution to capture fat tails of our daily financial return series, then, secondly, we use the theory of copula functions to measure and model the structure dependence between these extreme returns.

3.1. MODELING THE DISTRIBUTION TAILS OF FINANCIAL RETURNS AND GENERALIZED PARETO DISTRIBUTION:

To model the distribution tails of our stock returns and exchange rates, it is possible to use two principal models: Block Maxima model (BMM) and the Peak over Threshold model (POT).

This paper adopts the Peak over Threshold EVT method, which has several advantages: Indeed, it is quite flexible and realistic compared with the Block Maxima model, which does not take into account all the values susceptible to be extreme. Therefore, the BMM method uses annual or periodic maxima, which could lead to a loss of information contained in other large sample. However, the POT method avoids this problem by extracting the maximum over a threshold fixed in advance. The choice of threshold proves crucial for GPD approach. Indeed, good estimator quantile from a GPD model depends heavily on the choice of threshold.

In what follows, we retain the result of the Monte Carlo simulation made by De Melo Mendes (2005) Raggad [2007] and Marimoutou et al. [2009]. These authors have decided to retain always k = 100 (or 10% of 1000 cases used as estimation window). Accordingly, we retain u = 10%.

Now we follow the following procedure for estimating this approach:

- STEP 1: ESTIMATING A GARCH MODEL FOR THE STOCK RETURN SERIES AND EXCHANGE RATES IN THE MENA REGION:

The first step is to model our financial return series by the GARCH model using the Akaike information criterion (AIC). The GARCH model in this step severs to filter the return series such that GARCH residuals are closen to iid and gives the residuals for step-2.

- STEP 2: GPD FOR ESTIMATING THE QUANTILE RESIDUALS:

In this step, we choose the threshold value for each series of standardized residuals and we estimate the parameters of the Generalized Pareto distribution (GPD) by the method based on maximum likelihood.

We denote this approach by GARCH-GPD. This conditional approach offers several advantages: First, it captures the nature of our heteroscedastic time series studied through the GARCH model. Moreover, it allows a better description of extreme quantiles via EVT.

3.2. EXTREME COPULAS :

Recent empirical work has shown that most financial return series exhibit fat tail distributions and stochastic volatility. We also take into account that the bubbles (positive tails) or the crashes (negative tails) observed in the various markets are linked together. Z.-R. Wang et al., among others, showed that the Student copula and Clayton Archimedean copula best describe the dependency structure between several financial assets than the Gaussian copula.

4. EMPIRICAL RESULTS AND DISCUSSION:

4.1. DATA AND DESCRIPTIVE STATISTICS:

We use daily market data from twelve equity indices for the Middle East North Africa (MENA) countries, for a sample period of January 1, 2003 to December 31, 2012. We choose this period to investigate the impact of the 2007 Subprime crisis on the emerging countries of the MENA. The countries used in our sample are Bahrain (BHRALSH), United Arab Emirates (ABUGNRL), Kuwait (KWSEIDX), Oman (OMANMSN), Qatar (QTRMRKT), Saudi Arabia (TDWTASI), TUNISIA (TUNINDEX), MOROCCO (MASI), EGYPT (EGX30), ISRAEL (TEL AVIV 100), TURKEY (ISE100) and JORDON (AMMAN FM). We briefly overview summary statistics

TABLE 1: DESCRIPTIVE STATISTICS FOR STOCK EXHANGE ON THE COMPLETELY PERIOD STUDY

	Mean	Median	Maximum	Minimum	Standard Deviation
Saudi Arabia	0.015498	0.027019	7.122213	-5.073236	0.683392
Israël	0.011432	0.002168	4.218884	-4.576447	0.649481
Jordan	0.013424	0.00000	8.631166	-8.920678	0.518214
Kuweït	0.020996	0.025089	2.191839	-2.074734	0.365052
Turkey	0.017389	0.013789	23.47286	-29.10333	3.280671
Oman	0.010595	0.00000	4.299340	-4.782802	0.454114
Bahrain	0.000396	0.00000	1.569186	-2.136727	0.257211
Qatar	0.024578	0.014332	4.674500	-4.660230	0.648395
United Arab Emirates	0.013754	0.00000	17.29288	-15.84828	0.666830
Egypt	0.019291	0.00000	7.977635	-7.813663	0.769141
Morocco	0.009204	0.007477	1.978092	-2.178737	0.35487
Tunisia	0.018015	0.011816	1.784325	-2.173059	0.241665

From this table, we find that the mean daily stock returns ranges from 0.000396 to 0.024578 for Bahrain and Qatar. Volatility is measured by the standard deviation, which the lowest value is recorded on the Tunisian market (0.241390) and the largest on the Turkish market (3.280671).

TABLE 2: DESCRIPTIVE STATISTICS FOR EXCHANGE RATES ON THE COMPLETELY PERIOD STUDY

	Mean	Median	Maximum	Minimum	Standard Deviation
SAR / USD	-3.49^{e}-17	0.00000	0.598367	-0.184528	0.029211
ISL / USD	0.000974	0.00000	1.081287	-1.288287	0.164766
JOD / USD	1.42^{e}-05	0.00000	0.605447	-0.627950	0.080017
KWD / USD	0.000727	0.00000	1.038498	-1.257960	0.090864
TRY / USD	0.010919	0.00000	5.049033	-16.72436	0.441330
OMR / USD	2.20^{e}-06	0.00000	0.492067	-0.497083	0.036055
BHD / USD	-2.41^{e}-06	0.00000	0.874786	-0.876424	0.58224
QAR / USD	-2.86^{e}-16	0.00000	2.962594	-2.978401	0.074306
AED / USD	1.65^{e}-16	0.00000	0.317700	-0.269897	0.008992
EGP / USD	-0.005518	0.00000	4.799930	-6.780580	0.367981
MAD / USD	0.001652	0.00000	1.808426	-2.654072	0.415950
TND / USD	-0.001889	0.00000	6.571675	-6.439597	0.862391

According to the table above, we find that the mean daily exchange returns varies between -3, 49E-17 for Saudi Arabia and 0.001643 for Morocco. Volatility is measured by the standard deviation and including the lowest value recorded on the emirate market (0.008992) and the most important in the Tunisian market (0.862482). In addition, we note that the volatility of North -African markets recorded strong levels with a maximum of 0.862 for the Tunisian market contrary to the Middle East markets where they recorded a maximum of 0.58224 for the Bahrain.

4.2. Generalized Pareto Distribution Estimation:

The estimation results of the parameters of the Generalized Pareto Distribution (GPD) are reported in the following tables between the two sub-periods (pre-crisis and post-crisis) for stock returns and exchange rates. We recall that the parameters ζ, μ and σ are the shape parameters, scale and fixed threshold respectively. The value of the parameter ζ informs us about the weight of tails in the distribution.

- **STOCK RETURNS : Pre-crisis period : (2003-2007)**

Table 3: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: lower Tails

	TUNINDEX	EGX30	MASI	TDWTASI	AMMANFM	ABUGNRL
Ξ	-2.298	-3.510	-2.787	-2.671	-2.212	-3.876
Σ	2.972	8.794	8.876	7.148	6.080	11.208
M	-2.947	-4.189	-4.892	-4.622	-4.511	-4.578

	KWSEIDX	BHRALSH	QTRMRKT	ISE100	OMAN MSN	TEL AVIV100
Ξ	-2.599	-3.995	-3.017	-4.323	-2.947	-3.266
σ	8.673	12.199	7.256	0.075	9.578	9.063
μ	-6.160	-4.616	-4.167	-0.023	-4.793	-4.981

Table 4: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: upper Tails

	TUNINDEX	EGX30	MASI	TDWTASI	AMMANFM	ABUGNRL
ξ	-0.173	0.049	0.133	0.115	0.137	-0.053
σ	0.673	0.682	0.514	0.568	0.685	0.855
μ	1.720	1.577	1.600	1.428	1.525	1.69

	KWSEIDX	BHRALSH	QTRMRKT	ISE100	OMAN MSN	TEL AVIV100
ξ	0.014	0.073	0.066	0.996	0.157	-0.317
σ	0.893	0.863	0.556	0.001	0.797	0.980
μ	1.378	1.530	1.580	0.005	1.529	1.472

Concerning stock returns, the analysis of these results reveals that the estimated distribution parameter ζ is negative for the lower tail in all MENA stock markets indicating that the lower tails of these distributions returns are thin. However, it is positive for the right tail for all series of stock returns in the region (except for the Tunisian stock index (TUNINDEX) and Israel (AVIV100 TEL)). This observation indicates that the right tails of these distributions are thick, that is to say that the probability of occurrence of extreme gain is higher than required by the normal distribution. In addition, high positive returns are more likely than similar losses in these economies, before subprime crisis.

FOREIGN EXCHANGE RETURNS (2003-2007)

Table 5: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: lower Tails

	TND	EGP	MAD	SAR	JOD	AED
ξ	-5.486	-8.399	-2.339	0.604	-5.426	-1.077
σ	15.078	47.057	2.830	0.031	36.046	1.018
μ	-4.702	-7.239	-2.970	-5.278	-8.714	-4.073

	KWD	BHD	QAR	TRY	OMR	ISL
ξ	-7.784	-4.738	-2.755	-3.217	-3.468	-4.416
σ	54.64	17.601	11.046	13.868	19.959	11.562
μ	-8.888	-5.321	-6.26	-6.223	-7.447	-4.389

Table 6: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: upper Tails

	TND	EGP	MAD	SAR	JOD	AED
ξ	-0.087	0.058	-0.173	-0.047	0.605	-0.003
σ	0.442	1.006	0.722	1.695	0.167	1.726
μ	1.703	1.438	1.803	-0.180	1.293	0.469

	KWD	BHD	QAR	TRY	OMR	ISL
ξ	0.034	0.315	0.463	0.068	0.426	0.063
σ	0.660	0.859	0.486	0.561	0.641	0.524
μ	1.553	1.262	1.113	1.408	0.915	1.665

Regarding the foreign exchange returns, we observe that the estimated parameter ζ of the GPD seems negative for all lower tails of exchange returns studied (expect for SAR). This observation demonstrates the absence of left fat tails of these distributions returns. However, this shape parameter of the GPD appears positive for most of the upper tails of the returns indicating that the straight tails of these distributions are heavy and thick and a daily increase of the investor's capital is accompanied by a greater risk of extreme gains.

- **<u>STOCK RETURNS:</u> PRE-CRISIS PERIOD (2007-2012)**

Table 7: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: lower Tails

	TUNINDEX	EGX30	MASI	TDWTASI	AMMANFM	ABUGNRL
ξ	-6.047	-6.241	-3.340	-4.284	-5.111	-7.365
σ	31.69	33.283	9.474	19.446	20.034	60.24
μ	-6.959	-7.288	-4.646	-6.425	-5.665	-9.775

	KWSEIDX	BHRALSH	QTRMRKT	ISE100	OMAN MSN	TEL AVIV100
Ξ	-4.090	-3.871	-3.819	-3.282	-4.984	-3.160
Σ	22.27	13.019	14.131	7.962	20.764	8.901
M	-8.128	-5.291	-5.513	-4.236	-6.109	-4.913

Table 8: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: upper Tails

	TUNINDEX	EGX30	MASI	TDWTASI	AMMANFM	ABUGNRL
Ξ	0.364	0.070	0.063	0.128	-0.104	-0.051
Σ	0.425	0.363	0.618	0.488	0.675	0.547
M	1.525	1.357	1.500	1.320	1.514	1.337

	KWSEIDX	BHRALSH	QTRMRKT	ISE100	OMAN MSN	TEL AVIV100
Ξ	-0.006	-0.179	-0.053	-0.094	-0.099	-0.025
Σ	0.781	0.596	0.747	0.641	0.644	0.570
M	1.358	1.48	1.315	1.435	1.431	1.676

During the subprime crisis, the estimation values of the form parameter ζ for stock returns remain negative and decrease to the lower tail. This result suggests that the left tails of these distributions become less heavy than before the crisis. However, this parameter increased for the Egyptian stock market returns (EGX30), Saudi Arabia (TDWTASI) and Israel (TEL AVIV 100). In addition, the Tunisian stock return (TUNINDEX) becomes positive. As a result, the upper tails (right) distributions of these returns become thicker and heavier in the turbulent period. This finding informs investors that the probability of occurrence of extreme gains appears higher than required the normal distribution.

- **FOREIGN EXCHANGE RETURNS : (2007-2012)**

Table 9: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: lower Tails

	TND	EGP	MAD	SAR	JOD	AED
ξ	-7.906	-3.416	-3.737	-7.501	-3.774	-2.996
σ	44.018	4.999	8.772	27.091	15.599	1.204
μ	-7.171	-3.143	-3.840	-5.209	-5.927	-3.041

	KWD	BHD	QAR	TRY	OMR	ISL
ξ	-3.901	-4.369	-4.679	-2.560	-5.792	-3.328
σ	13.14	26.84	35.256	6.676	38.908	7.354
μ	-5.139	-7.820	-9.634	-4.49	-7.986	-3.887

Table 10: Estimation results of the GPD parameters (ζ, σ, μ) by the maximum likelihood method: upper Tails

	TND	EGP	MAD	SAR	JOD	AED
ξ	0.289	-0.514	-0.223	0.604	0.416	0.450
σ	0.544	1.388	0.759	0.292	0.443	0.912
μ	1.187	2.050	1.771	1.655	1.510	-0.228

	KWD	BHD	QAR	TRY	OMR	ISL
ξ	0.171	-0.104	0.238	0.132	0.310	0.059
σ	0.457	1.560	0.922	0.516	0.881	0.459
μ	1.631	1.016	0.855	1.469	0.998	1.694

Similarly, in order to study the evolution of these extremes, we conduct a comparison of the GPD ζ parameters for foreign exchange returns during the two sub-periods. The released results show that half of the foreign exchange returns have seen their parameters increased during the crisis including Egypt, Jordan, Kuwait, Bahrain, Turkey and Israel. Accordingly, the lower tails of these distributions become thicker after the crisis. In addition, Tunisia, Saudi Arabia, United Arab Emirates, Kuwait and Turkey exhibit additional extreme and the amplitude of these extremes appears larger in the second period than in the stable period reflecting a stronger probability of gains extremes. This finding remains interesting for financial investors who would likely see the possible change in the value of its portfolios under extreme event such as the subprime crisis.

4.3. ESTIMATES COPULA PARAMETERS:

4.3.1. FIT TEST COPULA:

The following table summarizes the result estimations of three information criteria (SIC, AIC and HQIC) having chosen the appropriate copula which model better extreme dependence co-movements of stock returns and foreign-exchange rates.

TABLE 11: ESTIMATES RESULTS OF FITNESS TESTS FOR STUDENT, CLAYTON AND GUMEBL COPULAS FOR LOWER TAILS DURING THE FIRST PERIOD (2003-2007)

RETURNS	STUDENT			GUMBEL			CLAYTON		
	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC
TUNISIA	129.17	132.95	131.63	311.15	314.93	313.61	440.48	444.27	442.95
MOROCCO	205.01	208.75	207.45	237.55	241.29	239.99	313.10	316.85	315.55
EGYPT	108.51	112.21	110.94	310.08	313.78	312.50			
SAUDI ARABIA	10.09	13.31	21.31	10.55	13.77	12.77	11.98	15.20	14.21
BAHRAIN	143.36	146.89	145.71						
UNITED ARAB EMIRATES	25.12	28.69	27.49	29.68	33.25	32.05	34.50	38.07	36.87
QATAR	57.21	60.86	59.61	67.21	70.87	69.61	80.83	84.04	82.78
KUWAIT	46.37	49.75	48.66	49.37	52.75	51.66	58.74	62.12	61.03
JORDAN	91.82	94.87	93.97	163.69	166.75	165.85	212.20	215.25	214.36 21
OMAN	153.91	156.78	156.00	-7.22	-4.35	-5.14			
ISRAEL	146.15	149.89	148.60	162.12	165.86	164.56	203.06	206.80	205.50
TURKEY	159.91	163.57	162.32	235.18	238.84	237.58	311.50	315.16	313.91

The reading of this table suggests that these information requirements are minimal in the case of Clayton copulas for the lower tails of the following countries: Tunisia, Morocco, Saudi Arabia, United Arab Emirates, Qatar, Kuwait, Jordan, Israel and Turkey. Egypt, meanwhile, chose the Gumbel copula that seems most appropriate. However, lower tails of financial returns from Oman and Bahrain seem better modeled by the Student copula.

TABLE 12: ESTIMATES RESULTS OF FITNESS TESTS FOR STUDENT, CLAYTON AND GUMEBL COPULAS FOR UPPER TAILS DURING THE FIRST PERIOD (2003-2007)

RETURNS	STUDENT			GUMBEL			CLAYTON		
	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC
TUNISIA	313.82	318.14	316.53	406.70	411.02	409.41			
MOROCCO	254.97	259.44	257.75	386.74	391.21	389.53	533.38	537.85	536.16
EGYPT	348.33	352.65	351.04	434.53	438.85	437.24			
SAUDI ARABIA	358.91	364.22	362.13	460.14	465.45	463.36	489.70	495.01	492.92
BAHRAIN	244.15	248.47	246.86						
UNITED ARAB EMIRATES	230.36	235.57	233.53	303.91	309.12	307.08	357.34	362.55	360.51
QATAR	254.43	259.47	257.50	332.86	337.90	335.94	423.65	428.69	426.73
KUWAIT	287.05	292.16	290.16	326.37	331.48	329.48	412.94	418.05	416.05
JORDAN	231.91	236.70	234.86	502.72	507.51	505.67			
OMAN	269.75	274.38	272.61						
ISRAEL	254.43	259.07	257.30	356.22	360.85	359.09	472.02	476.66	474.89
TURKEY	120.49	124.90	123.25						

The reading of this table provides information on selected copula to model the dependence between extreme stock and exchange returns in the MENA region. We observe that during the stable period, both the Student, Gumbel and Clayton copulas better model the dependence structure between the upper tails of these extreme returns.

TABLE 12: ESTIMATES RESULTS OF FITNESS TESTS FOR STUDENT, CLAYTON AND GUMEBL COPULAS FOR LOWER TAILS DURING THE SECOND PERIOD (2007-2012)

RETURNS	STUDENT			GUMBEL			CLAYTON		
	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC
TUNISIA	238.53	242.81	241.22	343.47	347.75	346.16	467.82	472.11	470.52
MOROCCO	290.80	295.12	293.51						
EGYPT	242.32	246.44	244.94						
SAUDI ARABIA	116.91	120.89	119.46	174.49	178.47	177.04	218.02	221.99	220.57
BAHRAIN	301.72	306.07	304.45	371.24	375.59	373.96	513.76	518.11	516.48
UNITED ARAB EMIRATES	4.92	9.07	7.55	1.90	6.06	4.53	1.69	5.84	4.32
QATAR	145.31	149.36	147.89	163.74	167.79	166.33	202.97	207.02	205.55
KUWAIT	117.92	122.17	120.60	136.68	140.93	139.36	166.45	170.70	169.13
JORDAN	181.46	185.61	184.09	398.38	402.54	401.02			
OMAN	335.74	339.96	338.40	401.05	405.27	403.71			
ISRAEL	199.51	203.73	202.17	243.06	247.28	245.72	311.35	315.57	314.02
TURKEY	302.03	306.08	304.61						

The reading of this table provides information on selected copula for each pair of extremes returns (stock market/ foreign exchange). We observe that the information criteria are minimal in the case of the Student copula for the following countries: Morocco, Egypt, United Arab Emirates and Turkey. For Jordan and Oman, both countries preferred the extreme Gumbel copula. The rest of the sample tends to bivariate Clayton copula.

TABLE 13: ESTIMATES RESULTS OF FITNESS TESTS FOR STUDENT, CLAYTON AND GUMEBL COPULAS FOR UPPER TAILS DURING THE SECOND PERIOD (2007-2012)

RETURNS	STUDENT			GUMBEL			CLAYTON		
	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC	-SIC	-AIC	-HQIC
TUNISIA	336.41	341.34	339.43	374.19	379.12	377.21	488.20	493.13	491.22
MOROCCO	343.76	348.39	346.63	444.39	449.02	447.25	628.50	633.14	631.37
EGYPT	231.51	236.15	234.38						
SAUDI ARABIA	349.00	353.93	352.02	562.16	567.09	565.18			
BAHRAIN	358.80	363.38	361.64	443.93	448.52	446.77	634.83	639.41	637.67
UNITED ARAB EMIRATES	325.74	331.46	329.19	257.00	262.72	2223.19	276.17	281.89	279.62
QATAR	163.05	168.22	166.20	213.59	218.76	216.74	325.94	331.11	329.08
KUWAIT	333.30	338.53	336.47	426.97	432.20	430.15	556.87	562.10	560.05
JORDAN	220.45	225.11	223.33	511.81	516.47	514.69			
OMAN	302.33	306.89	305.16						
ISRAEL	345.11	349.55	347.88	-8.61	-4.17	-5.84			
TURKEY	398.81	403.74	401.83	490.23	495.16	493.25	684.46	689.39	687.48

During the subprime crisis, the upper tails distributions of our returns appear best modeled by bivariate clayton copulas for Tunisian, Moroccan, Bahraini, emirates, Qatari, Kuwaiti and Turkey financial returns. The remaining countries in the MENA region seem best described both by the Gumbel copulas (Saudi Arabia and Jordan) and by bivariate student copulas (Egypt, Oman and Israel).

4.3.2. PARAMETER ESTIMATION OF SELECTED COPULAS:

To estimate the parameters of the selected copulas, we used the IFM (Inference Functions for Margins) which proposed by Shih and Louis (1995). First, we estimate the marginal parameters by the maximum likelihood method. Then, we introduce these estimators in the copula of the log-likelihood function to determine the copula parameter.

The table below shows the estimated parameters of copulas, for the lower extreme tails of MENA financial returns during the two sub- periods (before and after the crisis).

TABLE 14: ESTIMATED PARAMETERS COPULA: LOWER TAILS

LOWER TAILS							
PRE-CRISE (2003-2007)				**POST-CRISE (2007-2012)**			
COUNTRY	**COPULAS**	**PARAMETRES**	**COEFFICIENTS TAIL DEPENDENCE**	**COUNTRY**	**COPULAS**	**PARAMETRES**	**COEFFICIENTS TAIL DEPENDENCE**
EGYPT	Gumbel	ϴ = 91.86	0.992	**EGYPT**	Student	ρ = 0.95	
BAHRAIN	Student	ρ = 0.895		**BAHRAIN**	Clayton	α = 88.36	0.992
UNITED ARAB EMIRATES	Clayton	α = 1.30	0.588	**UNITED ARAB EMIRATES**	Student	ρ = 0.69	
ISRAËL	Clayton	α = 13.06	0.948	**ISRAËL**	Clayton	α = 21.27	0.967
JORDAN	Clayton	α = 32.66	0.979	**JORDAN**	Gumbel	ϴ = 138.6	0.994
MOROCCO	Clayton	α = 41.36	0.983	**MOROCCO**	Student	ρ = 0.972	
OMAN	Student	ρ = 0.97		**OMAN**	Gumbel	ϴ = 105.2	0.993
QATAR	Clayton	α = 3.02	0.794	**QATAR**	Clayton	α = 9.12	0.926
SAUDI ARABIA	Clayton	α = 0.71	0.376	**SAUDI ARABIA**	Clayton	α = 11.99	0.943
TUNISIA	Clayton	α = 139.42	0.995	**TUNISIA**	Clayton	α = 59.92	0.988
KUWAIT	Clayton	α = 2.54	0.761	**KUWAIT**	Clayton	α = 5.65	0.884
TURKEY	Clayton	α = 56.93	0.987	**TURKEY**	Student	ρ = 0.98	

Elliptical Student copula and Archimedean Gumbel and Clayton copulas are selected to understand the nature of dependency between extreme stockreturns and foreign exchange rates in MENA region.We find that the majority of pair financial returns (stock market/ foreign exchange) seem better modeled by the bivariate Clayton copula during the two periods of study. Indeed, we know that real returns tend to have larger lower tail dependence than the upper tail because the securities may crumble together rather than increasing together.

Moreover, we observe that all the estimated parameters are positive reflecting a positive dependence between these returns during periods of declining market and periods of rising market. However, the intensity of this dependence differs from one country to another and from one period to another and the released results depend on the selected copula.

At first, we begin with the estimated parameters of bivariate Clayton copulas, which have an asymmetric dependence structure and allow modeling only positive dependencies. Hence, they have only lower tail dependence and not the upper tail. Therefore, our interpretations are based on the values of tail dependence coefficients. This latter being between 0,995 recorded for Tunisia and 0,376 observed for Saudi Arabia. Thus, the presence of extreme co-movements and a highly lower tail distribution concentrated during the first period. Therefore, when the market is in bearish period, the dependence between stock and foreign exchange returns increases.

Gumbel Copula apprehends only positive dependencies and has asymmetrical upper tail dependence. During the first period, only Egyptian stock and foreign exchange returns seem modeled by this copula and record a tail value equal to 0.992. Thus, we can conclude that there are about a 99.2% chance that the Egyptian stock market returns realize a gain when foreign exchange returns registered positive returns.

Student copulas, meanwhile, have right and left tail dependencies, which are symmetrical. Therefore, upper and lower tail coefficients being similar. This symmetry could mean the same probability of occurrence of extreme events in the case of a bull market or a bear market. This is the case of stock returns and foreign exchange markets of Bahrain and Oman, which tend towards the bivariate Student copula.

To try to answer to the question of the changing in dependence structure and to compare the evolution of this dependence during the subprime crisis, we estimate the parameters of the bivairiate copulas between stock returns and foreign exchange rates in MENA region via the Inference Functions for Margins method. We find, as before, that globally the dependence structure between our extreme financial returns looks best modeled by the Clayton copula. However, we do not observe the same dependence strength for the two periods of study.

Indeed, we notice that the dependence parameter and the lower tail dependence coefficient increased for bivariate Clayton copula between stock and exchange returns for the following countries: Israel, Saudi Arabia, Qatar and Kuwait. Tails coefficients of these countries increased by 1.9%, 23.3%, 13.2% and finally 12.3% respectively. On the other hand, more this parameter is high, the dependence become greater. This means that these extreme returns are increasingly dependent after the occurrence of the subprime crisis. Consequently, an increase in the copula parameters across markets is adopted as a measure of the contagion effect. However, we observe a lower dependency between Tunisian stock returns and exchange rates that declined to 0.7% during the crisis.

Regarding the other remaining countries of the region, we detect existence of right and left tail dependence. Thus, there is a change in the dependency structure during the crisis. Indeed, for Egypt, United Arab Emirates, Morocco, Bahrain and Turkey, we observe that both the right and the left tail dependence become equal and symmetrical while it were asymmetrical in the first period.

With regard to Jordan, we see an increase of tail dependence coefficient during the crisis and a change in the dependence structure between its exchange rates and stock market returns. Indeed, the gains tail index becomes thicker than that of losses. This evolution of the dependence can have implications for strategies of an investor.

TABLE 15: ESTIMATED PARAMETERS COPULA: UPPER TAILS

UPPER TAILS							
PRE-CRISE (2003-2007)				**POST-CRISE (2007-2012)**			
COUNTRY	**COPULAS**	**PARAMETRES**	**COEFFICIENTS TAIL DEPENDENCE**	**COUNTRY**	**COPULAS**	**PARAMETRES**	**COEFFICIENTS TAIL DEPENDENCE**
EGYPT	Gumbel	Θ = 161	0.995	**EGYPT**	Student	ρ = 0.88	
BAHRAIN	Student	ρ = 0.93		**BAHRAIN**	Clayton	α = 134.93	0.9948
UNITED ARAB EMIRATES	Clayton	α = 12.305	0.945	**UNITED ARAB EMIRATES**	Student	ρ = 0.957	
ISRAËL	Clayton	α = 41.41	0.983	**ISRAËL**	Student	ρ = 0.97	
JORDAN	Gumbel	Θ = 83.13	0.991	**JORDAN**	Student	ρ = 0.86	
MOROCCO	Clayton	α = 82.09	0.991	**MOROCCO**	Clayton	α = 115.81	0.9940
OMAN	Student	ρ = 0.86		**OMAN**	Student	ρ = 0.95	
QATAR	Clayton	α = 17.16	0.960	**QATAR**	Clayton	α = 12.59	0.946
SAUDI ARABIA	Clayton	α = 14.47	0.953	**SAUDI ARABIA**	Gumbel	Θ = 118.83	0.9941
TUNISIA	Gumbel	Θ = 86.25	0.991	**TUNISIA**	Clayton	α = 29	0.976
KUWAIT	Clayton	α = 15.31	0.955	**KUWAIT**	Clayton	α = 28	0.975
TURKEY	Student	ρ = 0.61		**TURKEY**	Clayton	α = 92.021	0.992

Table 15 above summarizes the estimated parameter values of copula, for the upper extreme tails of financial returns during the two sub-periods (before and after subprime crisis).

As before, we find that the majority of pairs of financial returns (stock market / exchange rate) seem better modeled by the bi variate Clayton copula during the two periods of study. Another finding relates to the positivity of all the estimated parameters and coefficients of tail dependence of selected copulas improving the existence of a positive correlation between these returns during periods of declining market and periods of rising market. However, the intensity of this dependence differs from one country to another and from one period to another. We observe that both of Emiratis, Israelis, Moroccans, Qatari, Saudi and Kuwaiti stock market returns appear to have lower tail dependence with their respective foreign exchange rates. This implies that the stock returns of these countries in bearish period are likely to affect their foreign exchange returns. Based on the values of the lower tail dependence coefficients, we find that the Moroccan market records the thicker tail dependence, followed by the Israeli market with a

0.991 and 0.983 respectively. The lower tail dependence value is observed between stock and foreign exchange returns of the United Arab Emirates (0945). Stock returns of Bahrain, Oman and Turkey emerged, meanwhile, equal dependence in both distribution tails reflecting a balance between the right and the left tail upon the occurrence of an extreme event namely the subprime crisis. Thus, the dependence of these markets appears to be the same during up or down market periods. The highest correlation being recorded in the Bahraini market ($\rho = 0.93$), whereas the lowest correlation is observed between Turkey stock returns and exchange rates ($\rho = 0.61$). The dependence between extreme financial returns from Egypt, Jordan and Tunisia appears best described by extreme bivariate Gumbel copula reflecting a higher possibility of existence of joint extreme events during bullish periods markets rather than periods of bearish markets.

During the crisis, we find, as previously, that overall financial returns seem better modeled by the Clayton copula. However, we do not observe the same dependence strength for the two sub-periods. Indeed, we note that the lower tail dependence coefficients increased for bivariate Clayton copula between stock returns and exchange rates of Morocco and Kuwait. Thus, we observe an increase of 0.3% and 2% respectively. Consequently, the dependence between pairs of stock returns and exchange rates of these countries increased during the second period. This means that these extreme returns are increasingly dependent following the occurrence of the subprime crisis. However, we observe a lower dependency between stock returns and exchange rates of Qatar which declined by 1.4% during the crisis. The dependence structure of the Iranian market remains modeled by the Student copula during the crisis. Furthermore, this dependence changes after the subprime crisis and becomes higher. In addition, we detect a change in the dependency structure for the following countries: Egypt, United Arab Emirates, Israel and Jordan that exhibit similar and symmetrical right and left dependence coefficients.

5. CONCLUSION:

This paper examines the dynamics relationship between the MENA stock market and exchange returns after the occurrence of the financial subprime crisis, using daily data from January 2003 to December 2012. Based on copula and extreme value theory, the results during the pre-crisis period, show that the major of stock/exchange market returns have asymmetric dependence structure with asymmetric upper tail dependence. In addition, all estimated parameters are positive reflecting a positive dependence between these returns during periods of declining and rising market. However, the degree of the dependence becomes stronger when the financial crisis occurs and differs from one country to another.

Our main finding is the existence of the financial contagion effect between MENA stock market and exchange returns when the Subprimes financial crisis occurred. This finding is important for investors in their risk management during extreme market events.

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