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Quadratic Fractional Programming Problem–Special Case–Alternative Method

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ARTICLE INFO | ABSTRACT

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The paper provides a good alternative method for quadratic fractional programming problem (QFPP) concern with non-linear programming problem (NLPP) because the technique is useful to apply on numerical problems, reduces the labor work and save valuable time.

Non-linear Programming, Special case, Alternative method, Optimal solution, QFPP.

1. INTRODUCTION

The quadratic fractional programming problem (QFPP) studied with non-linear programming problem (NLPP) of maximizing (or minimizing) the quadratic fractional objective function in which a set of linear inequality constraints. In this, objective function is quadratic fractional and which is product of positive linear fractional functions. A single constraint problem is simple, but in the problem of more than two constraints or more in which complexities appear. In finance, the purpose of optimization technique is to find the optimum of a specific index number, usually the most favorable ratio of revenues and allocation. Therefore QFPP problem plays an important role in finance.

Mathematically, consider the special case of QFPP:

$$\text{Maximize } Z = \frac{(C_B x_{\boldsymbol{B}} + \alpha)(C'_B x_{\boldsymbol{B}} + \beta)}{(D_B x_{\boldsymbol{B}} + \gamma)(D'_B x_{\boldsymbol{B}} + \delta)}$$

Subject to the constraints: $Ax \leq b, x \geq 0$.

In this denominator is positive for all feasible solutions.

The above problem can be solved by starting with initial basic feasible solution then it improving the current basic feasible solution. Sharma S. D. [7, 8] used simplex method for solving such types of LPP, NLPP and dynamic programming. Charanes and Cooper [1] solved LFPP by equivalent linear programs. Khurana [4] studied such types of QFPP with linear homogeneous constraints. Nejmaddin [5] solved QFPP by Wolfe's and modified simplex method. Tantawy [9] used feasible direction method to solve QFPP and Salih [6] used linear fractional programming problems with extreme points. Hasan M. B. [3] also suggested some new technique to solve special type quadratic programming problems. Terlaky [10] gives an algorithm to solve QFPP but which does not require the enlargement of the basic table as Frank-Wolfe [2, 11] method. In addition, the special case of QFPP in which after converting the quadratic fractional objective function into the product of positive linear fractional functions will be solved by alternative method.

2. AN ALTERNATIVE ALGORITHM FOR SPECIAL CASE OF QUADRATIC FRACTIONAL PROGRAMMING PROBLEM

To find optimal solution of special case of QFPP by an alternative method, algorithm is given as follows:

Step 1. Check objective function of QFPP is of maximization. If it is to be minimization type then convert it to maximization.

Step 2. Convert quadratic fractional objective function to the product of linear fractional objective functions.

Step 3. Check whether all b_i (RHS) are non-negative. If any b_i is negative then convert it to positive.

Step 4. Express the given QFPP in standard form then obtain an initial basic feasible solution.

Step 5. Find net evaluations Δ_j for each variables x_j by the formula:

$$\Delta_j = \sum_{i=1}^{4} z^i \Delta_{ij}$$

where

$$z^1 = (C_B x_{\boldsymbol{B}} + \alpha), z^2 = \left(C_B' x_{\boldsymbol{B}} + \beta\right), z^3 = (D_B x_{\boldsymbol{B}} + \gamma), z^4 = (D_B' x_{\boldsymbol{B}} + \delta)$$
$$\Delta_{1j} = C_B x_{\boldsymbol{B}} - C_j, \Delta_{2j} = C_B' x_{\boldsymbol{B}} - C_j', \Delta_{3j} = D_B x_{\boldsymbol{B}} - D_j, \Delta_{4j} = D_B' x_{\boldsymbol{B}} - D_j'.$$

Step 6. Use usual simplex method for this table and go to next step.

Step 7. Check solution for optimality if all $\Delta_j \geq 0$, then current solution is an optimal solution, otherwise go to step 5 and repeat the same procedure. Thus optimum solution of special type of QFPP is obtained.

3. SOLVED PROBLEMS

Problem 3.1: Solve the following quadratic fractional programming problem:

$$\text{Maximize } Z = \frac{(x_1+2)(x_1+x_2+1)}{(x_1+2x_2+3)(x_2+3)}$$
$$\text{Subject to: } 4x_1 + 2x_2 \leq 8$$
$$x_1 + 2x_2 \leq 6$$
$$x_1, x_2 \geq 0.$$

Solution: Solving the above problem by an alternative method the detailed process of the solution is as follows.

QFPP is in standard form:

$$\text{Maximize } Z = \frac{(x_1+2)(x_1+x_2+1)}{(x_1+2x_2+3)(x_2+3)}$$
$$\text{Subject to: } 4x_1 + 2x_2 + s_1 = 8$$
$$x_1 + 2x_2 + s_2 = 6$$
$$x_1, x_2, s_1, s_2 \geq 0$$

where s_1, s_2 are slack variables.
By solving above problem, we obtain the following final simplex table:

Simplex table-1

	1	0	0	0
	1	1	0	0

						1	2	0	0	
						0	1	0	0	
C_B	C'_B	D_B	D'_B	BVS	X_B	x_1	x_2	s_1	s_2	$Ratio$
1	1	1	0	x_1	2/3	1	0	1/3	-1/3	
0	1	2	1	x_2	8/3	0	1	-1/6	2/3	
				z^1 = 8/3		0	0	1/3	-1/3	
	Z = 0.226			z^2 = 13/3		0	0	1/6	1/3	
				z^3 = 9		0	0	0	1	
				z^4 = 17/3		0	0	-1/6	2/3	
				Δ_j		0	0	2/3	40/3	

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is:

$x_1 = \frac{2}{3},\ x_2 = \frac{8}{3}$. Max. Z = 0.226.

Problem 3.2: Solve the following QFPP:

$$\text{Maximize } Z = \frac{8x_1{}^2+24x_1x_2+18x_2{}^2-2}{6x_1+9x_2+3}$$
$$\text{Subject to: } x_1 + 3x_2 \leq 5$$
$$2x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0.$$

Solution: Solving the above problem by an alternative method the detailed process of the solution is as follows.

QFPP is in standard form:

$$\text{Maximize } Z = \frac{(4x_1+6x_2-2)(2x_1+3x_2+1)}{6x_1+9x_2+3}$$
$$\text{Subject to: } x_1 + 3x_2 + s_1 = 5$$
$$2x_1 + x_2 + s_2 = 2$$
$$x_1, x_2, s_1, s_2 \geq 0$$

where s_1, s_2 are slack variables.
By solving above problem, we obtain the following final simplex table:

Simplex table-2

					4	6	0	0	
					2	3	0	0	
					6	9	0	0	
C_B	C'_B	D_B	BVS	X_B	x_1	x_2	s_1	s_2	$Ratio$
6	3	9	x_2	1.6	0	1	0.4	-0.2	
4	2	6	x_1	0.2	1	0	-0.2	0.6	
			z^1 = 8.4		0	0	1.6	1.2	
	Z = 2.8		z^2 = 6.2		0	0	0.8	0.6	
			z^3 = 18.6		0	0	2.4	1.8	
			Δ_j		0	0	2975.8	1255.4	

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is:

$x_1 = 0.2,\ x_2 = 1.6$. Max. Z = 2.8.

Problem 3.3: Solve the following programming problem:

$$\text{Maximize } Z = \frac{4x_1^2+12x_1x_2+8x_2^2+4x_1+4x_2}{4x_1+8x_2+4}$$

Subject to: $-2x_1 + x_2 \leq 3$
$4x_1 + 2x_2 \leq 8$
$x_1, x_2 \geq 0.$

Solution: Solving the problem by an alternative method the detailed process of the solution is as follows.

QFPP is in standard form:

$$\text{Maximize } Z = \frac{(2x_1+2x_2)(2x_1+4x_2+2)}{4x_1+8x_2+4}$$

Subject to: $-2x_1 + x_2 + s_1 = 3$
$4x_1 + 2x_2 + s_2 = 8$
$x_1, x_2, s_1, s_2 \geq 0$

where s_1, s_2 are slack variables.
By solving above problem, we obtain the following final simplex table:

Simplex table 3.3

					2	2	0	0	
					2	4	0	0	
					4	8	0	0	
$\boldsymbol{C_B}$	C_B'	D_B	BVS	X_B	x_1	x_2	s_1	s_2	$Ratio$
2	4	8	x_2	7/2	0	1	1/2	1/4	
2	2	4	x_1	1/4	1	0	-1/4	1/8	
			$z^1 = 7.5$		0	0	1/2	3/4	
	Z = 3.75		$z^2 = 11.5$		0	0	3/2	5/4	
			$z^3 = 33$		0	0	3	5/2	
			Δ_j		0	0	127.5	108.75	

Since all $\Delta_j \geq 0$, current solution is an optimum solution. Therefore optimum solution is:

$x_1 = \frac{1}{4}, \ x_2 = \frac{7}{2}$. Max. Z =3.75.

4. COMPARISION OF RESULTS:

Sr. No.	Quadratic Fractional Programming problem	Solution by Modified Simplex Method	Solution by Alternative Method
1	Maximize $Z = \frac{(x_1+2)(x_1+x_2+1)}{(x_1+2x_2+3)(x_2+3)}$ Subject to: $4x_1 + 2x_2 \leq 8$ $x_1 + 2x_2 \leq 6$ $x_1, x_2 \geq 0.$	$x_1 = \frac{2}{3}, x_2 = \frac{8}{3}$ Max. Z = 0.226	$x_1 = \frac{2}{3}, x_2 = \frac{8}{3}$ Max. Z = 0.226
2	Maximize $Z = \frac{8x_1{}^2+24x_1x_2+18x_2{}^2-2}{6x_1+9x_2+3}$ Subject to: $x_1 + 3x_2 \leq 5$ $2x_1 + x_2 \leq 2$ $x_1, x_2 \geq 0.$	$x_1 = 0.2, x_2 = 1.6$ Max. Z = 2.8	$x_1 = 0.2, x_2 = 1.6$ Max. Z = 2.8
3	Maximize $Z = \frac{4x_1{}^2+12x_1x_2+8x_2{}^2+4x_1+4x_2}{4x_1+8x_2+4}$ Subject to: $-2x_1 + x_2 \leq 3$ $4x_1 + 2x_2 \leq 8$ $x_1, x_2 \geq 0.$	$x_1 = \frac{1}{4}, x_2 = \frac{7}{2}$ Max. Z =3.75	$x_1 = \frac{1}{4}, x_2 = \frac{7}{2}$ Max. Z =3.75

In this table, compare the results of QFPP by alternative method and new modified simplex method. Here the value of objective function are same when it solved by alternative method after converting the quadratic fractional objective function to the product of positive linear fractional functions and when it solved by new modified simplex method.

CONCLUSIONS

An alternative method to the solution of special case of quadratic fractional programming problem (QFPP) has suggested. A number of algorithms have been developed to solve such types of QFPP, each applicable to specific type only. Our approach is general purpose to solve QFPP and reduce number of iterations by selecting pivot element also it gives more efficiency. The applications of QFPP are more and it is not possible to give a comprehensive survey of all of them. However, an efficient method to obtain the solution of QFPP is still. This technique is useful to apply on numerical problems, reduces the labor work and save valuable time.

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