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PERFORMANCES OF UTILITY BASED HEDGING AND EFFICIENT REHEDGING STRATEGIES TO OPTION REBALANCING

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ABSTRACT

One of the most successful approaches to obtain hedging with transaction cost is the utility based approach pioneered by Hodges and Neuberger (1989). Judging against the best possible trade off between the risk and cost of hedging strategy, this approach seems to achieve excellent empirical performance. However, the approach has one major drawback that prevents the broad application of it in practice, which is lack of rehedging function calibrated when the hedge ratio moves outside the prescribed tolerance. We overcome this draw back by presenting a simple efficient rehedging model and some other well known strategies and find that our model outperforms all others..

. **KEYWORDS**: Option hedging, Transaction cost, Utility based approach, Rehegding function

INTRODUCTION

A variety of approaches have been suggested to deal with the problem of option pricing and hedging with transaction costs. (See Mohamed (1994), Clewlow and Hodges (1997), Martellini and Priaulet (2002), and Zakamouline (2006). However, their numerical algorithm is cumbersome to implement and the calculation of the optimal hedging strategy is time consuming. According to the utility-based approach, the qualitative description of the optimal hedging strategy is as follows: do nothing when the hedge ratio lies within a so-called “no transaction region” and rehedge to the nearest boundary of the no transaction region as soon as the hedge ratio moves out of the no transaction region. One commonly used simplification of the optimal hedging strategy, widely used in practice, is known as hedging to a fixed bandwidth around delta. This strategy prescribes to rehedge when the hedge ratio moves outside of the prescribed tolerance from the corresponding Black-Scholes delta. Since there are no explicit solutions for the utility-based hedging with transaction costs and the numerical methods are computationally hard, for practical applications it is of major importance to use other alternatives. One of such alternatives is to caliberate a rehegding function when some parameters in the problem assume large or small values. Whalley and Wilmott (1997) were the first to provide this analysis of the model of Hodges and Neuberger (1989) assuming that transaction costs are small. Barles and Soner (1998) performed an alternative asymptotic analysis of the same model assuming that both the transaction costs and the hedger’s risk tolerance are small. However, the results of Barles and Soner (1998) are quite different from those of Whalley and Wilmott (1997). While Whalley and Wilmott (1997) derive only an optimal form of the hedging bandwidth which is centered around the Black-Scholes +delta, but with different delta specification .

In this paper, we implemented the optimal utility rehedging based model with a negative delta when the hedge ratio moves outside the prescribed region, providing the best fit to the exact rebalancing solution, making buy and hold strategies inefficient almost by default and comparing it with some other strategies with a positive delta within a unified utility based framework.

2.1 **THE METHOD**

THE UTILITY-BASED HEDGING METHOD

In modern finance it is customary to describe risk preferences by a utility function. The expected utility theory maintains that individuals behave as if they were maximizing the expectation of some utility function of the possible outcomes. Hodges and Neuberger (1989) pioneered the option pricing and hedging approach based on this theory. The key idea behind the utility based approach is the indifference argument: The writing price of an option is defined as the amount of money that makes the hedger indifferent, in terms of expected utility, between trading in the market with and without writing the option.
The starting point for the utility-based option pricing and hedging approach is to consider the optimal portfolio selection problem of the hedger who faces transaction costs and maximizes expected utility of his terminal wealth. The hedger has a finite horizon $[t;\ T]$ and it is assumed that there are no transaction costs at terminal time *T*. The hedger has the amount x_t in the bank account, and y_T shares of the stock at time *t*. We define the value function of the hedger with no option liability as

$$J_0(t\,, x_t, y_t, S_t) = \ maxE_t[U(x_T +\ y_T\ S\ _T)], \tag{1}$$

Where $$U(z) = U(x_T +\ y_T\ S\ _T).$$

Similarly, the value function of the hedger with option liability is defined by

$$Jw(t, x_t\ , y_t, S_t)\ =\ maxE_t[U(x_T\ +\ y_T\ ST -\ (ST -\ K)^+)]\,. \tag{2}$$

2.2 **THE HEDGING PROBLEM**

consider a continuous time economy with one risk-free and one risky asset, which pays no dividends. We will refer to the risky asset as the stock, and assume that the price of the stock, *St*, evolves according to a diffusion process given by

$$dSt\ =\ \mu Stdt\ +\ \sigma StdWt;$$

where μ and σ are, respectively, the mean and volatility of the stock returns per unit of time, and *Wt* is a standard Brownian motion. The risk-free asset, commonly referred to as the bond or bank account, pays a constant interest rate of $r \geq 0$. We assume that a purchase or sale of δS shares of the stock incurs transaction costs $\lambda|\delta S|$ proportional to the transaction ($\lambda \geq 0$).
We consider hedging a short option with maturity *T* and strike price *K*. We denote the value of the option at time *t* as $V\,(S_t, t\,)$. The terminal payoff of the option one wishes to hedge is given by

$$V\ (S_T, T\ )\ =\ max\ \{ST - K, 0\}\ =\ (S_T\ -\ K)^+\ , \tag{3}$$

The writing price is given as $$V\,(S_t, t\,) =\ Ke^{-\left(r+\frac{3}{2}\sigma^2\right)}. \tag{4}$$

The writing price of an option is defined as the amount of money that makes the hedger indifferent, in terms of expected utility, between trading in the market with and without writing the option.

As the stock price attains maximum $V(S_T, T) = Ke^{-\left(r+\frac{3}{2}\sigma^2\right)(T-t)}$. (5)

Recall that in the frame work of the utility based hedging approach, the option hedging strategy Is defined as the difference, $y_w(\tau) - y_0(\tau)$, between the hedger's optimal trading strategies with and without option liability. In the absence of transaction costs, the solutions for the optimal number of shares the hedger would hold without and with option liability are given by (see, for example, Davis et al. 1993)

$$y_0 = \frac{\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2}, \quad (6)$$

$$y_w = \frac{\delta(t,T)}{\gamma S}\frac{(\mu-r)}{\sigma^2} + \frac{\partial V}{\partial S}, \quad (7)$$

Where $\delta(t,T)$ is the discount factor given by

$$\delta(t,T) = e^{-r(T-t)}$$

Consequently, the option hedging strategy in the absence of transaction cost is simply the blackscholes strategy

$$\Delta = y_w - y_0 = \frac{\partial V}{\partial S}. \quad (8)$$

When a hedger writes an option, he receives the value of the option $V(S_t, t)$ and sets up a hedging portfolio by buying Δ shares of the stock and putting $V(S_t, t) - \Delta(1+\lambda)S_t$ in the bank account. As time goes, the writer rebalances the hedging portfolio according to some prescribed rule/strategy which are;

i) The unconditional sharpe ratio of the hedged portfolio at maturity which is the rebalancing ratio is given by

$$h(t_1) = \frac{V(S_t,t) - \Delta(1+\lambda)S_t}{U(z)}. \quad (9)$$

ii) The certainty equivalent growth rate of terminal wealth as measured by utility $U(z)$ is

$$U(Z) = -e^{(-\omega z)}; \quad \omega > 0. \quad (10)$$

$U(z)$ is the hedger's utility function and it is usually assumed that the hedger has a negative utility function.

Where ω is a measure of the hedger's (absolute) risk aversion. This choice of the utility function satisfies two very desirable properties:

(i) The hedger's strategy must not depend on his holdings in the bank account.

(*ii*) The computational effort needed to solve the problem must be relatively low.

(ii) Do nothing when the hedge ratio lies within a so-called "no transaction region" and rehedge to the nearest boundary of the no transaction region as soon as the hedge ratio moves out of the no transaction region.

This particular choice of utility function might seem restrictive. However, as it was conjectured by Davis et al. (1993) and showed in Andersen and Damgaard (1999), an option price is approximately invariant to the specific form of the hedger's utility function, and mainly only the level of absolute risk aversion plays an important role.

3.1 OTHER MODELS

Using lelands method one hedges an option with the delta of modified price calculated in accordance with the formula (9), but with adjusted volatility. (Allaneda and Paras .1994)

$$\delta m = \sigma\left(1 + \frac{2\lambda}{\sigma\sqrt{\delta t}}\right) \quad (11)$$

Where $\delta m = \sigma^2$

$$\Delta = \frac{\partial V(\delta m)}{\partial S} = N(d_1(\delta m))\,, \quad (12)$$

$$d_1 = -\sigma\sqrt{T-t}\ . \quad (13)$$

,

Whalley and Wilmott (1993) and Avellaneda and Paras (1994) derived an adjusted volatility as

$$\delta m = \sigma\left(1 + \sqrt{\left(\frac{8}{\pi\delta t}\right)\frac{\lambda}{\sigma}}\, sign(\Gamma)\right) \quad (14)$$

With
$$\Gamma = \frac{N^{'}(d_1)}{S\sigma\sqrt{(T-t)}} \quad (15)$$

And
$$\Delta = N(d_1) \pm \left(\frac{3}{2}\,\frac{e^{-r(T-t)}\lambda S T^2}{\gamma}\right)^{\frac{1}{3}}. \quad (16)$$

Where $N(d_1) = \frac{\partial V}{\partial S}$

With application of equation (9)

Black-Scholes hedging strategy consist in holding Δ shares of the stock and some amount in the bank account using equation (9) . (Black and Scholes . 1973)

With
$$\Delta = \frac{\partial V}{\partial S} = N(d_1)\,, \quad (17)$$

Where

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}\ . \quad (18)$$

4.1 OUR MODEL

Theorem

Consider hedging an option with maturity T and strike price K. the value of the option at time t is $V(S_t, t)$. If the maximum point is $V(S_T, T)$ and the utility $U(z)$ follow a fractal function $(W(z))^\gamma$ then the optimal strategy of the terminal option payoff one wishes to hedge has the Sharpe ratio given by

$$h(t_2) = \frac{K^\gamma}{-\gamma\omega\Delta\alpha}\ , \quad (19)$$

where γ is the rehedging function.

Proof

Using harmonized fractal dimensional model (Osu and Ogwo, 2015). We have

$$h(t_2) = \frac{1}{\Delta\alpha}\int_0^\infty (W(z))^\gamma dz\ . \quad (20)$$

Where $$W(z) = V(S_T, \text{T})\,(U(z)) \tag{21}$$

$V(S_T, T) = Ke^{-\left(r+\frac{3}{2}\sigma^2\right)(T-t)}$, from equation (5)

$U(Z) = -e^{(-\omega z)}$; $\gamma > 0$. From equation (10)

Let $(T-t) = \mu$ and $\left(r + \frac{3}{2}\sigma^2\right) = x$

We have

$$h(t_2) = \frac{1}{\Delta\alpha}\int_0^{\infty} (Ke^{-\left(r+\frac{3}{2}\sigma^2\right)(T-t)})\left(-e^{(-\omega z)}\right))^{\gamma} dz.$$

Therefore

$$h(t_2) = \frac{1}{\Delta\alpha}\int_0^{\infty} (Ke^{-(x)\pi})\left(-e^{(-\omega z)}\right))^{\gamma} dz$$

$$h(t_2) = \frac{K^{\gamma}}{\Delta\alpha}\int_0^{\infty} (-e^{-((x)\pi+\omega z)})^{\gamma} dz$$

$$h(t_2) = \frac{K^{\gamma}}{\Delta\alpha}\int_0^{\infty} (-e^{-\gamma((x)\pi+\omega z)}) dz$$

let $u = -\,\gamma\big((x)\pi + \omega z\big),$

$\frac{du}{dz} = -\gamma\omega$

$dz = \frac{du}{-\gamma\omega}$

$$h(t_2) = \frac{K^{\gamma}}{\Delta\alpha}\int_0^{\infty} (-e^{u}) dz$$

$$h\,(t_2) = \frac{K^{\gamma}}{\Delta\alpha}\int_0^{\infty} (-e^{u})\frac{du}{-\gamma\omega}$$

$$h(t_2) = \frac{K^{\gamma}}{-\gamma\omega\Delta\alpha}\int_0^{\infty} (-e^{u})\, du$$

$$h(t_2) = \frac{K^{\gamma}}{-\gamma\omega\Delta\alpha}, \tag{19}$$

as required.

4.2 PERFOMANCES OF $h(t_1)$ and $h(t_2)$

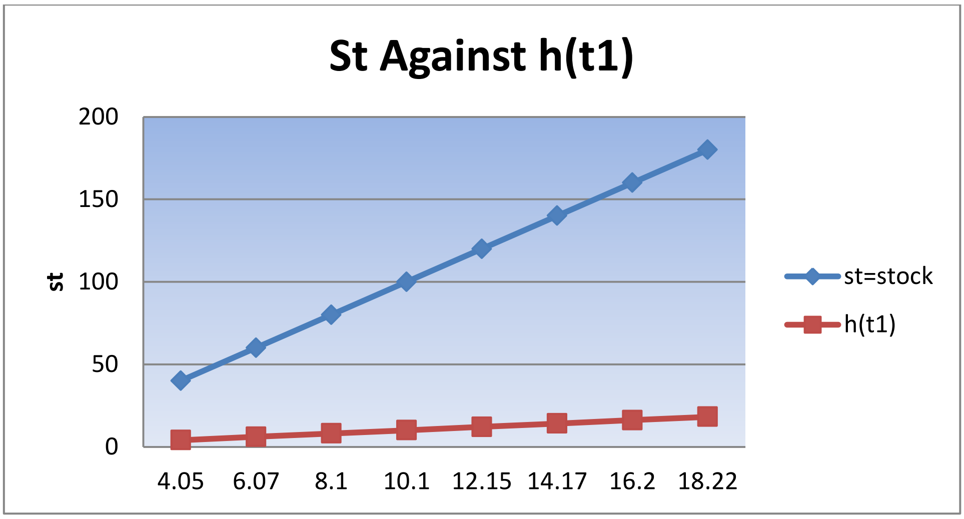
The optimal hedging strategy (6) with positive delta versus (16) with negative delta for the following model parameters: $\gamma = 0.25,\ \lambda = \omega = 0.01,\ S_t = K = 40{,}60{,}80{,}100{,}120{,}140{,}160{,}180,\quad \alpha = 100, \sigma = 0.25, \mu = r = 0.05, and\ T - t = 0.5.$ Zakamouline (2006).

Using equation (6) with $+\Delta$ and equation (16) with $-\Delta$ we have the table below.

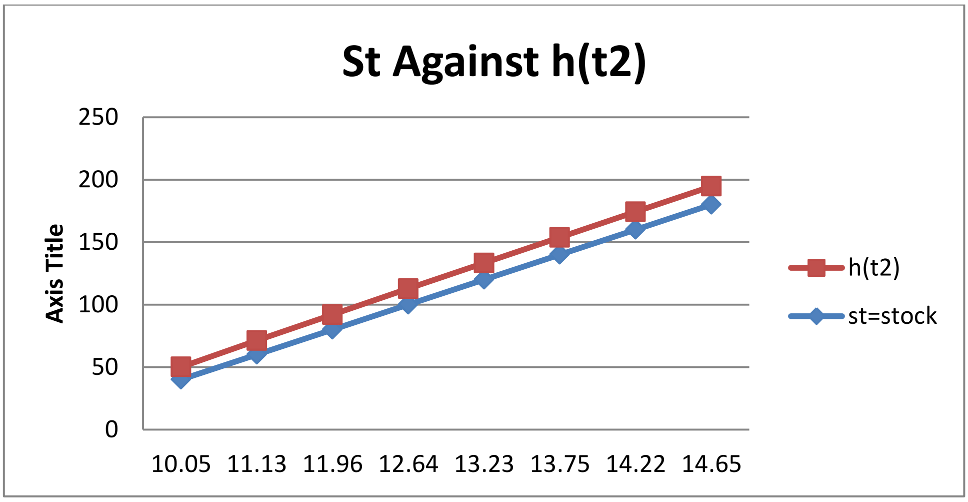
S_t = stock price	$h(t_1)$ Hedging boundaries with $+\Delta$	$h(t_2)$ Hedging boundaries with $-\Delta$
40	4.05	10.05
60	6.07	11.13
80	8.10	11.96
100	10.1	12.64
120	12.15	13.23
140	14.17	13.75
160	16.20	14.22
180	18.22	14.65

$Gragh\ of\ S_t$ Against $h(t)$)

Figure 1



Here the hedging boundary follow the trend of the stock price hence has no range.
Figure 2



Here, the hedging boundary has range and can be predicted

Conclusion

In practice, option can either be settled in asset or cash and the type of option settlement affects the option price and hedging strategy. Here, our model assumption is made for simplicity either with cash or asset and the hedgers strategy does not depend on his holding in the bank account. To this, the computational effort needed to solve the problem is low, hence it examine the optimality that preserve hedging coefficient without excessively compromising banks overall efficiency as the hedging boundary has range and can be predicted.

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