Short Notes

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Iterates of maps: Stability A Mathematical Modle in Life Sciences

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ABSTRACT

One of the features which starts to emerge is the question of equilibrium points, and the question of whether they are stable or unstable. It must be clear to you that this is a matter of very considerable practical importance; if we are studying a system, perhaps a biological one involving the functioning of an organ in the body, or the concentrations of reagents in a cell, perhaps an economic one involving inflation, the difference between a system which runs wild and one which settles down is usually more important than the details of just how long it takes to do either of these things.

KEYWORDS: equilibrium points, economic, concentrations

INTRODUCTION

As remarked, this is only a small part of what mathematicians have learnt over the years, but this is a course on mod- elling, not on difference or differential equations as such. And it represents a healthy start. Quite a lot of interesting systems can be modelled to a useful extent by such equations.

One of the features which starts to emerge is the question of equilibrium points, and the question of whether they are stable or unstable. It must be clear to you that this is a matter of very considerable practical importance; if we are studying a system, perhaps a biological one involving the functioning of an organ in the body, or the concentrations of reagents in a cell, perhaps an economic one involving inflation, the difference between a system which runs wild and one which settles down is usually more important than the details of just how long it takes to do either of these things. This is the qualitative theory of difference or differential equations, and it complements the numerical solution of equations in an important way. Again, there is a huge amount known about these matters, and we shall only skip over the surface. The subject was started seriously in the last years of the last century, so it goes back about one hundred years, and has evolved into Topology sometimes known as Rubber-Sheet Geometry, (which turns up everywhere these days). I tell you this sort of thing because there are more important things in life than exams, and getting some sort of perspective on which way things are going is one of them.

You can think of stability as studying the question of what happens if you wobble things a little bit. It sounds classier if we talk of the effects of per- turbations on systems, but it means the same thing. we shall look at stability in the simplest cases and get a clearer picture of what to look for and what kinds of things can happen.

Cobwebs and Chaos

Going back to first order equations, when I ask you to calculate the first ten terms of the sequence which solves

$y(n+1)=my(n)+c; y(0)=d$

what you did was to start off by putting $n = 0$ to get $y(1) = md+c$. Then you took the answer to this calculation and fed it back into the same process to get out the next value.

A useful way to look at this is to think of the function $y = mx + c$

and to put in a value for x, $y(0)$ and see it as outputting $y = y(1)$. Then we get the next value by feeding the output back to the input. The stream of numbers we get out is the solution sequence. This is, of course, a very easy thing to put on a computer.

This works with other first order difference equations too; if we had the non-linear logistic approximation equation

$y(n + 1) = y(n) + my(n)(a - y(n))\Delta$

we could just take the function

$f(x) = x + m(x)(a - x)\Delta$

and again feed the output back into the input. Thinking of a function as an input-output device is very useful, and thinking of this feedback process as generating a sequence of numbers is also very useful.

There is a graphical way of doing this called a cobweb diagram which has been much used by economists, and we shall see why shortly. Let me explain the idea of a cobweb diagram for the simplest sort of equation first.

The difference equation I shall look at first is $y(n + 1) = 3y(n)(1 - y(n)); y(0) = 1/2$ (4.1)

If we start off by writing down the iteration map $y = 3(x)(1 - x)$

and initialise our sequence generator with $x = 1/2$, we get $1/2, 3/4, 9/16, 189/256, \cdots$

and it is hard to see any particular structure to what is going on.

If we run this on a suitable computer program, we see that the system oscil- lates and seems to settle down to about 2/3 which looks as though it could be an equilibrium point. If so it is a fixed point of the iteration map; we simply say that

$x = 3x(1 - x)$ when x is a fixed point and solve to get

$x = 3x - 3x^2$ or $3x^2 = 2x$ giving $x = 0$ and $x = 2/3$. So there are exactly two fixed points

of this function. We can get a better picture of what is going on by drawing the graph of

$y = 3x(1-x)$ and looking to see where it cuts the graph of $y = x$. We get the cobweb by starting at input 0.5. We go up to the curve which

is the graph of the iteration function. This gives the output on the vertical

axis. To turn it back into an input we move horizontally until we hit the line $y = x$. Now we move vertically until we hit the curve, and so on. It is now obvious from the diagram that 2/3 is a stable fixed point of the iteration. You can see the 'moving point' homing in by a sort of square spiral towards the fixed point.

If we do a numerical plot of the solution to the differential equation, we get something like figure 4.2

With a bit of thought you can see the relation between the two plots: the oscillations in figure 4.2 as it settles down to 2/3 are the projections on the Y-axis of the points on the cobweb.

Many students want to know the one true way of thinking about things, and if Mathematics has any generally important thing to say, it is that there IS no one right way. On the contrary, the more ways of looking at something the better.

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